

202/232

B.C.A. (Part - II) EXAMINATION, 2021

(Faculty of Science)

(Three - Year Scheme of 10+2+3 Pattern)

DISCRETE MATHEMATICS

Time Allowed : Three Hours

Maximum Marks : 100

No supplementary answer-book will be given to any candidate. Hence the candidates should write their answers precisely in the main answer-book only.

All the parts of one question should be answered at one place in the answer-book. One complete question should not be answered at different places in the answer-book.

Write your roll number on question paper before start writing answers of questions.

PART - I : (Very short answer) consists of 10 questions of 2 marks each. Maximum limit for each question is upto 40 words.

PART - II : (Short answer) consists of 5 questions of 4 marks each. Maximum limit for each question is upto 80 words.

PART - III : (Long answer) consists of 5 questions of 12 marks each with internal choice.

PART - I

Attempt all parts of the question.

1. (a) Let $a \equiv b \pmod{x}$ and y be any integer then show that $a - y \equiv b - y \pmod{x}$.
- (b) Expand $(1 + x)^5$ using Binomial theorem.
- (c) If $A \subseteq B$ then show that $A \oplus B = B - A$.
- (d) Define equivalence relation.
- (e) Prove that $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$.
- (f) Let $\langle B, +, \cdot, 0, 1 \rangle$ be a Boolean algebra, then for all $a \in B$, prove that $a + a = a$.
- (g) Define simple graph.
- (h) Define product of two graphs.
- (i) Define Tree.
- (j) Define Spanning Tree.

PART - II

Attempt all the parts of the question.

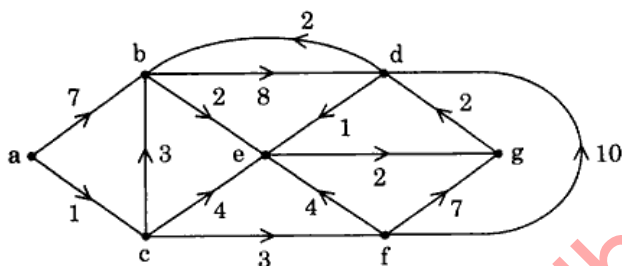
2. (a) Solve $a_r = a_{r-1} + a_{r-2}$; $r \geq 2$, $a_0 = 0$, $a_1 = 1$.
- (b) If A, B, C and D are any four sets, then prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
- (c) If p and q are two statements then show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.
- (d) Prove that the numbers of edges in a simple graph with n vertices and k connected components ($k \geq 1$) cannot exceed $\frac{(n-k)(n-k+1)}{2}$.
- (e) Prove that there is one and only path between every pair of distinct vertices in a tree.

PART - III

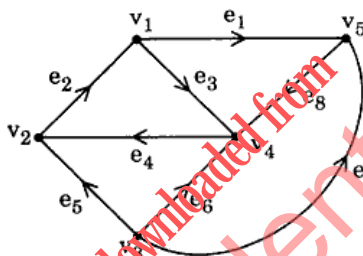
Attempt all questions by taking any two parts from each question.

3. (a) Prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n.
- (b) Find the Co-efficient x^r for the generating function $G(x) = \sum_{r=0}^{\infty} a_r x^r = \frac{x^2 - 5x + 3}{x^4 - 5x^2 + 4}$.
- (c) Solve the recurrence relation $a_r - 6 a_{r-1} + 9 a_{r-2} = r \cdot 3^r$
4. (a) How many integers are there between 1 and 1000 which are not divisible by 2, 3, 5 or 7?
- (b) Is the relation $R_1 = \{(a, b) | ab + 1 > 0 ; a, b \in R\}$ on the set R of real numbers, equivalence relation? If not, explain.
- (c) Prove that the inverse of a one-one onto function is one-one, onto.
5. (a) Prove by means of truth table, that $p \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$
- (b) In the Boolean algebra $\langle B, +, \cdot, ', 0, 1 \rangle, \forall a \in B$, prove that $(a')' = a$.
- (c) Prove that, no Boolean Algebra can have exactly three distinct elements.

6. (a) Find the shortest path between the vertices a and g in the following directed weighted graph.



- (b) If G is simple connected planer graph with n vertices and e edges ($e > 2$), then $e \leq 3n - 6$.
- (c) Find the adjacency matrix and the incidence matrix of the following directed graph.



7. (a) If T is binary tree with n vertices and of height h, then prove that $h + 1 \leq n \leq 2^{h+1} - 1$.
- (b) Prove that a graph G is connected if and only if it has a spanning tree.
- (c) Discuss Kruskal's algorithm to find a minimal spanning tree for a weighted connected graph.

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